Reg. No. : | | | | | | | | | | | |

Question Paper Code : 21303

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2018.

Second Semester

Civil Engineering

MA 6251 — MATHEMATICS — II

(Common to Mechanical Engineering (Sandwich), Aeronautical Engineering, Agriculture Engineering, Automobile Engineering, Biomedical Engineering, Computer Science and Engineering, Electrical and Electronics Engineering, Electronics and Communication Engineering, Electronics and Instrumentation Engineering, Environmental Engineering, Geoinformatics Engineering, Industrial Engineering, Industrial Engineering and management, Instrumentation and Control Engineering, Manufacturing Engineering, Materials Science and Engineering, Mechanical Engineering, Mechanical and Automation Engineering, Mechatronics Engineering, Medical Electronics Engineering, Petrochemical Engineering, Production Engineering, Robotics and Automation Engineering, Biotechnology, Chemical Engineering, Chemical and Electrochemical Engineering, Fashion Technology, Food Technology, Handloom and Textile technology, Information Technology, Petrochemical Technology, Petroleum Engineering, Pharmaceutical Technology, Plastic Technology, Polymer Technology, Textile Chemistry, Textile Technology, Textile Technology (Fashion Technology))

(Regulations 2013)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Show that $\nabla\phi$ is a vector perpendicular to the surface $\phi(x, y, z) = c$ where c is a constant.

2. Evaluate $\int_C [(x-2y)\,dx + (3x-y)\,dy]$, where C is the boundary of a unit square.

3. Find the particular integral of $(D^2+4)y = x^4$.

4. Transform the differential equation $x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+4y=\log_e x$ to a differential equation with constant coefficients.

5. Find the Laplace transform of $\int_0^t \left(u^2-u+e^{-u}\right)du$.

6. Find : $L^{-1}\left[\frac{1}{(s-2)^4}\right]$.

7. Verify whether $u=(x-y)\left(x^2+4xy+y^2\right)$ is harmonic function.

8. Define the cross ratio of four points in a complex plane.

9. Classify the singularity of the function $f(z)=z^2e^{1/z}$ at $z=0$.

10. Find the poles of the function $f(z)=\frac{z^3}{\cos z}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the angle between the surfaces $x^2-y^2-z^2=11$ and $xy+yz-zx=18$ at the point (6, 4, 3). (8)

(ii) Find the work done by the force $\overline{F}=\left(2xy+z^3\right)i+x^2j+3xz^2k$, when it moves a particle from (1, –2, 1) to (3, 1, 4) along any path. (8)

Or

(b) (i) If $\overline{v}=\overline{\omega}\times\overline{r}$, prove that $\overline{\omega}=\frac{1}{2}\text{curl }\overline{v}$, where $\overline{\omega}$ is a constant vector and $\overline{r}=xi+yj+zk$. (8)

(ii) Evaluate $\iint_S \overline{F}\cdot\hat{n}\,dS$, where $\overline{F}=4xzi-y^2j+yzk$ and S is the surface of the cube defined by $x=0$, $x=1$, $y=0$, $y=1$, $z=0$, $z=1$. (8)

12. (a) (i) Solve $(D^2+16)y = 2e^{-3x} + \cos 4x - 1$. (8)

(ii) Solve $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = x^2 + \frac{1}{x^2}$. (8)

Or

(b) (i) Solve $y'' - 4y' + 4y = (x+1)e^{2x}$ by the method of variation of parameters. (8)

(ii) Solve the equations $2\frac{dx}{dt} + x + \frac{dy}{dt} = \cos t$, $\frac{dx}{dt} + 2\frac{dy}{dt} + y = 0$. (8)

13. (a) (i) Find the Laplace transform of the periodic function $f(t) = \begin{cases} t, & 0 \le t \le a \\ 2a - t, & a \le t \le 2a \end{cases}$ and $f(t+2a) = f(t)$. (8)

(ii) Find the inverse Laplace transform of $\frac{14s+10}{49s^2+28s+13}$. (8)

Or

(b) (i) Using convolution theorem, find $L^{-1}\left\{\frac{1}{s^2(s+1)^2}\right\}$. (8)

(ii) Solve the initial value problem $y'' - 6y' + 9y = t^2 e^{3t}$, $y(0) = 2$, $y'(0) = 6$, using Laplace transform. (8)

14. (a) (i) If $u = x^2 - y^2$ and $v = -\frac{y}{x^2+y^2}$, prove that both u and v satisfy Laplace equations, but $u + iv$ is not an analytic function of z. (8)

(ii) If $f(z) = u + iv$ is an analytic function of z in any domain, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^p = p^2|f(z)|^{p-2}|f'(z)|^2$. (8)

Or

(b) (i) Find the image of the circle $|z| = 2$ under the transformations $w = z + 3 + 2i$ and $w = \sqrt{2}e^{i\pi/4}z$. (8)

(ii) Find the bilinear transformation that maps the points $1+i, -i, 2-i$ of the z- plane into the points 0, 1 , i of the w – plane. (8)

15. (a) (i) Represent the function $f(z) = \dfrac{4z+3}{z(z-3)(z+2)}$ in Laurant's series.

(1) when $0 < |z| < 1$,

(2) when $1 < |z| < 2$ and

(3) when $|z| > 2$. (10)

(ii) If $g(a) = \int_C \dfrac{3z^2 + 7z + 1}{z-a}\, dz$, where C is the circle $|z| = 2$, find the values of $g(3)$, $g'(1-i)$ and $g''(1-i)$. (6)

Or

(b) (i) Evaluate $\int_0^\infty \dfrac{x \sin x}{x^2 + a^2}\, dx$, by contour integration. (10)

(ii) Evaluate $\int_C \dfrac{\cos(\pi z^2) + \sin(\pi z^2)}{(z+1)(z+2)}\, dz$, where C is $|z| = 3$. (6)